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AN ESSAY

ON

MECHANICAL GEOMETRY,

CHIEFLY EXPLANATORY OF A SET OF

SCHEMES and MODELS,

BY WHICH

The knowledge of the moſt uſeful propoſitions of EUCLID, and other celebrated Geometricians, may be clearly and expeditiouſly conveyed, even to youth of an early age.

By ***BENJAMIN DONNE,***
Maſter of the Mechanics in Ordinary to
His MAJESTY.

K

Quicquid in aſtronomicis, geographicis, vel nauticis efficiendum; geometriæ attribuendum eſt.

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1796.

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BRITISH MUSEUM

TO

THE MOST NOBLE

JAMES

MARQUIS of SALISBURY,

KNIGHT OF THE GARTER,

LORD CHAMBERLAIN OF HIS

MAJESTY's HOUSEHOLD, *&c.*

This Attempt to render the Introduction to the

important Science of

GEOMETRY

Both perſpicuous and entertaining, is in gratitude

for favours received, reſpectfully inſcribed,

by

His Lordship's

moſt humble and

obedient Servant,

BENJAMIN DONNE.

Bristol, September 12, 1796.

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LETTER

FROM

Dr. BEDDOES TO Mr. DONNE.

SIR,

Clifton, Auguſt 18, 1796.

MULTIPLIED occupations of the moſt urgent nature put it totally out of my power to furniſh you at preſent with thoſe preliminary remarks on the utility of your mathematical models, which I gave your ſubſcribers reaſon to expect. I do not however think any reflections I could offer of ſuch importance, as to make it worth your while to ſuſpend the delivery of your boxes; and I really know not when I ſhall be leſs engaged.

I intended to ſtate, that you by no means ſtand pledged to my theory of mathematical evidence. Neither have I been active in bringing out your mechanical demonſtrations, as ſuppoſing them calculated to corroborate my arguments.

ments. Thoſe arguments muſt ſtand by their own ſtrength. I do not ſuppoſe them likely to be ſpeedily overthrown.

Your apparatus in my opinion will be of infinite uſe to parents, intereſted in ſecuring to their children the bleſſing of a clear and juſt underſtanding. A much greater proportion, I believe, than nine out of ten of thoſe, who are educated to the profeſſions, or to live without a profeſſion, conceive an inſuperable diſguſt againſt Geometry, as it is uſually taught: and very little management will moſt ſurely be ſufficient to prevent this pernicious effect by the help of your models.

It may be thought, that the long demonſtrations in Euclid are of uſe in beſtowing a facility in conceiving and recalling long chains of argument. This advantage I ſhall not call in queſtion; for I am not diſpoſed to depreciate the merits of the antient Geometricians.

I ſhall however obſerve that, as all ideas are derived from ſenſe, all argument muſt conſiſt of a ſtatement of facts or perceptions. The true way therefore of making ideas durable, or rather eaſily excitable, is to make them diſtinct at firſt. It was on this account truly obſerved, that "*the* "*art of memory is the art of attention.*" The ſame end will be anſwered by any contrivance, calculated to render perceptions vivid. On this principle your tangible proofs of the properties of figures will be eminently ſerviceable to the intellectual faculties of young people. The effect will be exactly the reverſe of that produced by READING-MADE-EASY and by the GRAMMARS in uſe.

For theſe ſeveral years, I have been correſponding and converſing with different friends about a project, much allied to that which you have now executed, and which will come in very well after yours. It is to eſtabliſh a manufacture of RATIONAL TOYS. I believe parents are become ſufficiently attentive to education to give ſuch a ſcheme ſupport; and fortunately it cannot alarm any prejudice. The deſign is to conſtruct models, at firſt of the moſt ſimple, and afterwards of more complicated machines. The models are not to be very ſmall, and they are to be ſo conſtructed, that a child may be able to take them to pieces, and to put them together again. The particulars of the deſign are too numerous to be given here. It comprehends engravings and a good deal of letter-preſs. I have in view, not merely information in mechanics, chemiſtry, and technology, but the improvement of the ſenſes by preſenting, in a certain order and upon principle, objects of touch along with objects of ſight. In this important buſineſs we have hitherto truſted to chance. But there is every reaſon to ſuppoſe, that INTELLIGENT ART will produce a much quicker and greater effect. Should inſtruction addreſſed to ſenſe be made in any country the principle of education, ſhould the beſt methods of cultivating the ſenſes be ſtudied, and ſhould proper exerciſes be deviſed for reproducing ideas (originally well-defined) ſometimes with rapidity, at others in diverſified trains, the conſequence is to me obvious. The inhabitants of that country would ſpeedily become as far ſuperior to the reſt of mankind in intellect and efficiency—in the SCIRE and POSSE of Bacon—as the moſt cultivated people of Europe are now ſuperior to the Portugueſe, or to the Moors of Barbary.

The deſign which I have here intimated is as boundleſs as nature and art. In the courſe of the enſuing winter I hope to get a few ſets of theſe *rational toys*, with the engravings and the explanation, executed.

Others may proceed upon that foundation; or, without waiting for me, they may deviſe a plan for themſelves upon the hints I have here thrown out.

I am, Sir, with all good wiſhes,
Your very obedient ſervant,
THOMAS BEDDOES.

PREFACE.

THE utility of GEOMETRY is ſo well underſtood, that it is only neceſſary to delineate the particular character of the preſent work. The aſpect of Mathematical Science, as it is commonly exhibited, cannot be denied to be forbidding. Experience evinces that an exceedingly ſmall proportion of thoſe, who are made to attend Lectures on the Mathematics, as eſſential to a liberal education, become maſters of the firſt ſix books of Euclid. Perſons leſs fortunately circumſtanced, ſeldom have opportunity in early life to learn the properties of plane and ſolid figures; and yet they commonly feel a curioſity to obtain ſome inſight into Natural Philoſophy, and the various Arts, in which the principles of Geometry find their application.

Theſe conſiderations induced the Author, about thirty years ago, to invent *mechanical*, or *palpable* demonſtrations of the moſt important propoſitions in Geometry. From 1766 to the preſent time, he has repeatedly exhibited them in London, Briſtol, and other places. In the ſame way he has exemplified ſome of their practical uſes in Navigation, Architecture, taking of Heights, Menſuration, Gauging, and Surveying. And he has enjoyed the ſatisfaction of being able to render the truths he wiſhed to communicate, quickly, and diſtinctly intelligible. Some other Lecturers, he has reaſon to think, have ſince partially adopted his plan; and it is poſſible that ſimilar ideas may have occurred to various perſons; but the ſeries of demonſtrations, now preſented to the public, is entirely his own. Their utility, he preſumes, will be acknowledged; though nothing of the kind, as far as he knows, has ever been offered to the public.

In the Proſpectus it was promiſed that the Apparatus ſhould conſiſt of upwards of 50 ſchemes and models in card-paper, wood, and metal: to render it more uſeful, it has been conſiderably extended. By it may be conveyed to very young perſons the knowledge of the fundamental propoſitions in Geometry, as well thoſe of Euclid as ſome others which do not occur in that author. To acquire mathematical information will be rendered by this contrivance an amuſement inſtead of a taſk. The repugnance generally excited by the ordinary method will be avoided, and proficiency in the exact Sciences will be much expedited, by ſo advantageous an introduction. Nor can the Apparatus be accounted dear, as by it more propoſitions may be taught in an Hour than in a Week by Euclid, or any other Treatiſe of abſtract Geometry. The Author is not ſingular in believing that his Work will be found highly uſeful to all Tutors, whether in private Families, in Boarding Schools or Academies, and even in the Univerſities themſelves.

When the Proſpectus of this Work was firſt publiſhed, it was not intended to touch on the *Fifth Book* of Euclid, which treats of the doctrine of proportion; partly on account of its difficulty, and partly from its not being a ſubject adapted to mechanical proofs. However, as ſome of the propoſitions are too valuable to be paſſed by in ſilence, an *Eighth Book* is added, in which it is preſumed they are explained in a manner ſufficiently clear to be readily underſtood by young Students; if they have already acquired a general knowledge of the Rule of Three, in common arithmetic.

The Author cannot conclude this Preface without thanking his very reſpectable Subſcribers for their liberal ſupport: to ſeveral he is indebted, not only for their ſubſcription, but for the active part they have taken in recommending the work; more particularly he muſt acknowledge his obligations to Thomas Beddoes, M. D.*; as probably the work would never have been publiſhed, had it not been for his preſſing perſuaſion, and uncommon activity, in recommending the plan to his literary friends.

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* This Gentleman, without any previous knowledge of the Author's invention, had inſiſted upon the neceſſity of teaching the Elements of Geometry on this Plan, in a work entitled "Obſervations on demonſtrative Evidence" publiſhed by Johnſon, St. Paul's Church-Yard.

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N. B. The Author flatters himſelf, that thoſe Subſcribers whoſe names have not been tranſmitted, would rather excuſe their non-inſertion, than that the Publication ſhould be delayed longer, by waiting for them.

An ESSAY *on*

MECHANICAL GEOMETRY.

BOOK I. of

DEFINITIONS, ANGLES, & TRIANGLES.

DEFINITIONS.

GEOMETRY* is the ſcience of extenſion; which treats of the properties of lines, angles, ſurfaces, and ſolids.

2. *A Phyſical Point* is an indefinitely ſmall quantity, as a dot (.) made with a point of a needle. But a *Mathematical* or *Geometrical Point* is not a quantity, but only a term or bound of a quantity: Or, in Euclid's words, is "that which hath

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* According to its etimology, ſignifies the art of meaſuring land; being probably the firſt or principal uſe, to which the knowledge of geometry was applied.

no part or magnitude. See *ſcholium*, Article 42.

3. A *Line* may be conceived to be generated by the motion of a point. If the point in motion moves continually in the ſame direction, it deſcribes a *right* or *ſtraight*
C. 1. *line;* as a line drawn by the ſide of a ſtraight ruler; and is the neareſt diſtance between its two extreme points.

4. But, if the point in motion be continually changing its direction, it then deſcribes a crooked line, and is named a
F. 2. *Curve.*

Note, For the future, whenever we uſe the term a line, we mean a *right line*, unleſs otherwiſe particularly expreſſed.

5. A *Solid*, in a geometrical ſenſe, is any thing that has length, breadth, and thickneſs, whatever may be its form; whether it be a ſolid in the common acceptation of the word, as a piece of wood or ſtone; or hollow, as a box, or glaſs-tumbler.

6.

Note, The marginal references are to the cards and figures; thus, C. 1. C. 2. &c. direct to the Number of the card; and F. 1. F. 2. &c. to the Number of the ſcheme in the card, if more than one.

6. The bounds of a ſolid are called *Superficies* or *Surfaces*, and have only length and breadth, but no thickneſs.—Surfaces may be either plane, convex, or concave.

7. A *Plane Surface* or *Superficies* is a flat ſurface; or that on which if we lay a ſtraight ruler, in any direction whatever, it will touch it in every part. Such is the upper ſurface of a well-planed table.

8. A *Convex Surface* is a ſurface that riſes upward, as the outward ſurface of a chryſtal of a watch.

9. A *Con~~vex~~*cave *Surface* is a hollow ſurface, as the inſide ſurface of a chryſtal of a watch.

10. *Surfaces* or *Superficies* may be of any ſhape; for as ſolids may have any form, regular or irregular: ſo the ſuperficies which bound the ſolids, muſt vary with the figure of the ſolids.

11. If two right lines meet each other in a point (ſo as not both together to become a right line) the poſition of thoſe two lines, with reſpect to each other, is called a *Rectilineal Angle*. Euclid ſays, it is the inclination of two ſtraight lines. Thus the incli-

C. 1. F. 3. F. 4. inclination of two lines BA, CA, is an angle. But if one line ſtands upright on the other, as the lines DE, FE, one cannot be ſaid to incline to the other; and yet it is by all Geometricians called a *Right Angle*, and conſequently Euclid's definition is not ſufficiently general.

F. 5. 12. If an angle opens more than a right angle, it is called an *Obtuſe Angle*.

13. If an angle opens leſs than a right angle, it is named an *Acute Angle*. F. 5.

14. If there are two or more angles meeting at the ſame point, it is common to expreſs any particular angle by three letters, of which the middle one is at the vertex, or point of meeting, and the other two at the lines forming the angle. Thus F. 6. DBC, or CBD, ſignifies the angle at B, formed by the meeting of the two lines DB, CB. And ABD, or DBA, the angle at B, formed by the lines AB, DB.—Again, ABC denotes the angle formed by the meeting of the lines AB and CB. But if there is but one angle at a point, it is generally expreſſed by a ſingle letter at that point.

15. If a pair of compaſſes be opened to any extent, and the point of one of its legs at reſt in the point C, (on a plane ſurface) and the other leg turned round, the point of it will deſcribe a curve line, every where equidiſtant from the point C, which point is called the *Center;* and the ſpace contained by that curve line a *Circle.*— Any line drawn from the center to the curve, the *Radius.** — The curve line itſelf, or the *Periphery,* is named the *Circumference.* It is manifeſt from this generation of a circle, that all lines drawn from the center to any part of the circumference are equal; and that a line paſſing from any point of the curve through the center, and produced 'till it comes to the circumference on the oppoſite ſide, is equal to double the radius; which line is named a *Diameter.* Any part of the circumference of a circle is called an *Arch,* or as now more generally written, an *Arc.* C. 2. F. 1.

16. The equality or inequality of angles is beſt determined by arcs of equal circles:

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* All the lines iſſuing from a center to the circumference, like the ſpokes in a coach-wheel, are called *Radii.* In Card 5, a number of them are drawn.

C. 2. F. 2. 3 Thus, figures 2 and 3, CAB and *cab*, repreſent two angles. With any extent of a pair of compaſſes, reſting one point in A, figure 2, ſweep the arc *de*, and with the ſame extent, and one point in *a*, figure 3, ſweep the arc *m n*, then taking with the compaſſes the extent from *d* to *e*, figure 2, remove the compaſſes, and ſetting one point in *m*, figure 3, ſee if the other point will reach exactly to *n*; if it does, the angles are manifeſtly equal; if not, it will diſcover which is the greateſt, and their difference.

17. Hence Mathematicians, for the more eaſy comparing and calculating the proportion which angles bear to each other, ſuppoſe the circumference of every circle to be divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, named *ſeconds*, &c.

N. B. Sometimes the circumference of a circle is called a circle; as when it is ſaid a circle is divided into 360 degrees, &c.

18. When one line ſtands exactly upright C. 2. F. 4. on another, it is called a *Perpendicular*, as the line DC. And the angle ACD being equal to the angle BCD, as a ſemicircle

is equal to 180 degrees (the whole being 360) each angle muſt contain an arc of the half of 180 degrees, viz. 90 degrees. Hence we ſay a *Right Angle* is equal to 90 degrees; an *Obtuſe Angle* is greater than 90 degrees; an *Acute Angle* leſs than 90 Degrees.

19. A *Rectilineal Figure* is any figure bounded by right lines.

20. A *Plane Triangle** is a figure bounded C. 2.
by three right lines. F. 5.

21. The lines bounding a triangle are named *Sides*.

22. An *Iſoſceles Triangle* has two ſides C. 3.
equal. F. 1.

23. An *Equilateral Triangle* has three equal ſides. F. 2.

24. A *Right-angled Triangle* is that which F. 3.
has one right angle; or which is the ſame, has one of its ſides perpendicular to the other, as the ſide BC is upright, or perpendicular to the ſide AB, the ſide AB is called

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* We mention a *Plane Triangle* above to diſtinguiſh it from a triangle, bounded by arches of circles, on a Globe; which belongs to Spherical Geometry, and comes not within the deſign of the preſent eſſay. Therefore in this eſſay by a triangle we always mean a plane triangle.

called the *Baſe;* BC the *Perpendicular*, and the ſide AC, oppoſite to the right angle, the *Hypothenuſe*.

25. An *Acute-angled Triangle* is any triangle which has all three angles acute.

26. An *Obtuſe-angled Triangle* is that which has one of its angles obtuſe.

27. An *Oblique Triangle* is a general Name for all triangles which have not a right angle.

28. A *Scalene Triangle* is any triangle which has three unequal ſides.

29. If a triangle has three angles equal to the three angles of another triangle, each to each, they are called *Similar Triangles*.

30. Any figure bounded by four lines is called *quadrilateral*.

C. 3. 31. A *Square* is a figure which is bounded
F. 4. by four equal lines, and has four right angles.

F. 5. 32. An *Oblong*, *Rectangle*, or *Right-angled Parallelogram*, is a figure contained under four lines, whoſe oppoſite ſides are equal, and hath four right angles; but not four equal ſides.

33. A *Rhombus* is a figure having four equal ſides; but its angles are not right angles. C. 3. F. 6.

34. A *Rhomboid*, or *Rhomboides*, is a four ſided figure, whoſe oppoſite ſides are equal to each other, but not all four equal, nor are its angles right angles. C. 4. F. 1.

35. All other four-ſided figures are called *Trapeziums*, and a line drawn from any angle to its oppoſite angle is called a *Diagonal*; thus the figure ABCD repreſents a Trapezium, and the lines AC, DB are its *Diagonals*. F. 2.

36. *Multilateral Figures*, or *Polygons*, are figures of any ſhape, which are bounded by more than four lines; if all the ſides are equal they are called *Regular Polygons*.

Note, The term *multilateral* denotes many ſides, and *Polygon* many angles; but if a figure has many ſides, it muſt have many angles.

37. If any part of a circle is divided or cut off by a line, the part cut off, be it a large or ſmall part, is called a *Segment*, and the line which divides the circle is named the *Chord*, or baſe line of the ſegment:

C. 4. thus, the chord line AB divides the circle
F. 3. into two unequal ſegments, ABD and AEB; but the chord line *a b* paſſing through the center C, becomes a diameter, and divides the circle into two equal ſegments, *aDb*, and *aEb*, and therefore each is called a ſemicircle.

38. If two lines are drawn from the center of a circle to two points in the circumference, cutting off a part leſs than a ſemicircle, the part ſo cut off is named a
F. 4. *Sector*, as ABC.

39. *Parallel Lines* are thoſe which are every where at the ſame diſtance from each other, and therefore if they could be infinitely produced, would never meet. Thus AB,
F. 5. CD, the diſtances *ab*, *cd* being equal, are parallel lines.

C. 3. F. 4. 5
40. A *Parallelogram* is a four-ſided figure,
6. & whoſe oppoſite ſides are parallel, whether
C. 4. F. 1. the angles be right angles or not.

41. A *Cube* or *Die* is a ſolid figure bounded
F. 6. by ſix equal ſquares, erected perpendicular to each other.

42. *Scholium*,* We have taken notice,

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* Scholium ſignifies a remark or note.

that Euclid ſays a point is that which has no part or magnitude; to this it hath been objected, "that if it has no part or magnitude, it can have no exiſtence." Perhaps we may be able to clear this difficulty, by obſerving that, in the figure which repreſents a cube, the common place of meeting of the three planes ABGF, BCDG, FGDE, is at G; C. 4.
and therefore the point G, the common ter- F. 6.
mination of the three planes is really a mathematical point; it manifeſtly has no part or magnitude, but muſt exiſt with the ſolid, though it is no part of it.

In like manner according to Euclid, a line is length without breadth, and therefore its exiſtence has been denied, as well as that of a mathematical point: But the two planes ABGF and FGDE terminate or meet each other in the line FG, which muſt have exiſtence with the ſolid, though no part of it.

The definitions of other ſolids, &c. will be given hereafter as occaſion may require.

43. Mathematicians make uſe of the terms, poſtulates, axioms, propoſitions, problems, and theorems.

A POSTULATE is ſomething required to be granted, to prevent cavils. Euclid's poſtulates are,

1.

Let it be granted, that a ſtraight line may be drawn from any one point to any other point,

2.

That a terminated ſtraight line may be produced to any length, in a ſtraight line.

3.

That a circle may be deſcribed from any center, at any diſtance from that center.

To theſe of Euclid ſhould be added another poſtulate, viz.

4.

Grant that one figure may be laid on another.

See note on this poſtulate in our Geometry, article 304, page 8. 111

44. AXIOMS are truths ſo exceedingly clear, as to require no demonſtration. In Euclid we find the following:

1.

Things which are equal to the ſame are equal one to another.

2.

If equals are added to equals, the wholes are equals.

3.

If equals be taken from equals, the remainders are equals.

4.

If equals be added to unequals, the wholes are unequals.

5.

If equals be taken from unequals, the remainders are unequals.

6.

Things which are double of the ſame, are equal to one another.

7.

Things which are halves of the ſame, are equal to one another.

8.

Magnitudes which coincide with one another, that is, which exactly fill the ſame ſpace, are equal to one another.

9.

The whole is greater than its part.

10.

Two ſtraight lines cannot incloſe a ſpace.

11.

All right angles are equal to one another.

Note, The 12th of Euclid is not ſufficiently clear to be called an axiom, but requires a demonſtration; and has been the cauſe of much debate amongſt both the ancient and modern Geometricians; for which reaſon it is here omitted, and a more uſeful one inſerted, viz.

12.

All the parts taken together are equal to the whole.

Note, This is not in Euclid, though he frequently makes uſe of it.

45. PROPOSITIONS are things propoſed to be done, and are of two kinds, Problems and Theorems. A PROBLEM is a practical propoſition, as *to draw one line perpendicular to another*. A THEOREM, a ſpeculative propoſition, the truth of which may be demonſtrated; as *the three angles of a triangle are together equal to two right angles, or* 180 *degrees*.

46. A Demonstration is the proof of the truth of any aſſertion.

47. Though it be our principal deſign in this eſſay to convey clearly the knowledge of the moſt uſeful *Theorems*, with their *eaſy* or *mechanical proofs*, yet it may be expected that we ſhould give ſome of the moſt uſeful *Problems* before we proceed; but as their rationalé could not be underſtood without knowing the propoſitions on which they depend, we ſhall firſt proceed with the Theorems, and afterwards give ſome of the moſt neceſſary Problems.

BOOK I. SECTION II.

Of angles made by lines meeting, or croſſing each other.

Theorem I. *Any angle, if meaſured by a large or ſmall circle, will be found to contain the ſame number of degrees.*

For as a degree, by article 17th, is the 360th part of the circumference of any circle, be it a large or ſmall circle, the length of a degree muſt increaſe or decreaſe in the ſame proportion as the circumference of the circle increaſes or decreaſes; and conſequently the arc repreſenting any number of degrees C. 5.

degrees in one circle, will be to the whole circumference, as the arc repreſenting the ſame number of degrees in any other circle to the whole circumference of that other circle. Hence it follows, that if the angle DCB by the large circle contains any number of degrees, for example, 60; it will interſect the ſame number of degrees on the ſmall circle.

Hence the reaſon of expreſſing the magnitude of an angle, by ſaying it contains a certain number of degrees.

THEOREM 2. *A Line ſtanding on another line making angles with it, the ſum of theſe angles is equal to two right angles, or* 180 *degrees.* 13 E. 1. or 4 D. 1.*

The line DC with the line AB makes two angles ACD and BCD. About the point of interſection C, with any radius (extent of the compaſſes) ſweep the ſemicircle *adb*: Now the arc *bd* is the meaſure of the angle BCD, and the arc *ad* the meaſure of the angle ACD; but both theſe arcs make up the ſemicircle, or 180 degrees, equal to two right angles.

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* 13 E. 1. or 4 D. 1. is to be read the 13th propoſition of Euclid's 1ſt. Book, or 4th theorem of Donne's 1ſt. Book; and the like for others.

COROLLARY. *Hence if one of the angles be a right angle, the other muſt alſo be a right one.*

THEOREM 3. *If two or more lines ſtand on the ſame line, at the ſame point, making angles with it, the ſum of all the angles is equal to two right angles, or* 180 *degrees.* Corollary 2. of D. 1.

For let two lines DC, EC be drawn, C. 7. meeting in the line AB in the point C, and about C as a center with any convenient radius deſcribe the ſemicirle *adcb*; then is the arc *bc* the meaſure of the angle DCB, arc *cd* of the angle DCE, and the arc *da* the meaſure of the angle ECA; but theſe three arcs compleat the ſemicircle *adcb*, or the three angles are equal to 180 degrees, or two right angles. And hence it muſt alſo follow, that if ever ſo many lines met in the point, the angles muſt be equal to the ſemicircle, or 180 degrees.

THEOREM 4. *If two or more lines interſect one another in the ſame point, all the angles taken together are equal to* 4 *right angles, or* 360 *degrees.* Corollary 4 of 4 D. 1.

C.8. Thus if two lines AE, DB interſect or croſs each other in the point C; then about C, as a center, with any convenient radius deſcribe a circle *abcd*, and it is manifeſt that the meaſures of the 4 angles *a*C*d*, *d*C*c*, *c*C*b*, and *b*C*a*, are together equal to 360 degrees; and if ever ſo many croſs each other in the ſame point, all the angles taken together can make no more than the whole circle, or 360 degrees.

THEOREM 5. *If two lines interſect each other, the oppoſite angles are equal.* Thus C.9. the angles ACD and BCE are equal; alſo Part 1 the angles DCB and ACE are equal to each other. 15 E. 1; or 15 D. 1.

For as the lines AB and DE are made to Part 2 turn about the center C, as much as you open one angle upward you will open the other downward, and therefore in every poſition the oppoſite angles formed muſt be equal to each other.

Scholium. In ſome ſchemes there are dots made to catch the eye; thus if two angles have one dot in each (.) it denotes that they are equal to each other; thus is the angle ACD equal to the angle ECB. If two angles are re each marked with two dots (..) it ſhews they are equal; alſo, if two angles are marked with three dots (·.·) it ſignifies that they alſo are equal to each other. Again, if in two ſchemes, as for inſtance in Card 12, in the two triangles, if one ſide of one is croſſed with a daſh (|) and one ſide of the other triangle is alſo marked with a daſh, it denotes that theſe ſides are equal; and if another ſide in each triangle is marked with two daſhes (||) it ſhews that theſe two ſides are equal. Laſtly, if the remaining ſide of one is croſſed with three daſhes (|||) and the remaining ſide of the other is alſo marked with three daſhes, it denotes alſo an equality between them.

THEOREM 6. *If a line croſſes two parallel lines the adjacent angles are equal; that is, thoſe on the ſame side are equal to each other.— Thus, let AB be parallel to CD, EH a line* C. 10 *croſſing them; then I ſay the angle EFB (in which I have made one dot (.) is equal to the adjacent angle FGD (in which I have alſo made one dot* (.) 28 E. 1. or 7 D. 1.

For AB being parallel to CD, and EF and FG being one and the ſame right line; it is evident EF muſt have the ſame inclination or poſition with reſpect to FB, as FG has to GD; which is all that is to be underſtood by angles being equal.

THEOREM 7. *If a line croſſes two parallel lines, the alternate angles are equal; that is, the angle on one side of the croſſing line is equal to the angle on the other side, viz. Let AB and CD be two parallel lines, and EH a line croſſing them; then the angle AFG (in which is made one dot* (.) *is equal to the alternate angle FGD, (in which is alſo made a dot* (.) Def. 32. D. or 20 E. 1.

For if the angle ~~EFB~~ AFG is cut out, it will coincide with the angle FGD; but this will be ſufficiently clear without cutting the figure. For the angle AFG is equal to the angle EFB by Theorem the 5th; and the angle FGD is alſo equal to the angle EFB by Theorem 6th; therefore being equal to one and the ſame thing they muſt be equal to each other.

BOOK I. SECTION III.

Of TRIANGLES.

Theorem 8. *If two triangles ABC, DEF, have two sides, and their contained angle in one, equal to two ſides, and their contained angle in the other, viz. the side AB* C.12 *equal to DE, AC equal to DF, and angle A equal to angle D, theſe triangles are equal in every reſpect.* 4 E. 1, or 1 D. 1.

For DE being equal to AB, if we cut out the triangle DEF (or it will be ſufficiently clear merely to conceive it done) the line DE may be placed on AB, the point D on A, and the point E on B; then the angle EDF, being equal to the angle BAC, the line DF will fall on the line AC; and the point F on C; and conſequently the line FE on CB; and ſo the triangle EDF will exactly coincide and cover the triangle BAC, and therefore muſt be equal in every reſpect.

Theorem 9. *The angles at the baſe of an iſoſceles triangle are equal. Thus the triangle ABC having the two ſides AC and BC* C.13 *equal, the angle A will be equal to the angle B.* 5 E 1, or 2 D. 1.

For biſect the angle C by the line CD; that is, let the angle ACD and angle BCD be equal to each other; then to prove this Theorem mechanically, cut out the triangle DCB, and laying it on the triangle DCA, it will exactly coincide with, or cover it; therefore the angles A and B are equal.

Corollary. If the angle C of an iſoſcles triangle be biſected, the biſecting line will be perpendicular to the Baſe, viz. CD perpendicular to AB.

THEOREM 10. *Equilateral triangles are equiangular.*

C. 14 For the three ſides of the triangle ABC being equal, AC is equal to BC; therefore the angle A is equal to the angle B by Theorem the 9th, and BC being equal to BA, by the ſame Theorem the angle C is alſo equal to the ſame angle A, and conſequently all three are equal; or it might be proved mechanically, were it neceſſary, by cutting out one of the angles, and placing it on each of the others, it will coincide and ſhew their equality.

THEOREM 11. *Any two sides of a triangle are greater than the third.* 20 E. 1, or 6 D. 1.

Thus

the sum of
Thus, the two leſſer ſides AC and BC of C. 15
the triangle ABC is greater than the longeſt ſide AB. This is ſo evident that *Proclus* derides Euclid for giving a formal demonſtration. For would any traveller in his ſenſes who wanted to go from the town A to the town B in the neareſt road, go firſt to the town C, and from thence to the town B? Certainly he would travel on the direct road AB, as the neareſt diſtance to any two points is a right line; and therefore if he deviates out of it, he muſt go farther; or, in other words, AC and CB taken together muſt be greater than AB.

THEOREM 12. *If a triangle has unequal sides, the greateſt side is opposite to the greateſt angle; a leſſer side to a leſſer angle, and the leaſt side to the leaſt angle.* 19 E. 1, or 12 D. 1.

About the angular points A,B,C, with any C. 16
radius deſcribe the arcs *ab*, *cd*, *ef*, and by inſpection, or meaſuring with a pair of compaſſes, it appears that the arc *dc*, the meaſure of the angle C, which is oppoſite to the greateſt ſide AB, is greater than the arc *ef*, the meaſure of the angle B, which

is oppoſite to the leſſer ſide AC; and the arc *ab*, the meaſure of the angle A, is the leaſt, and is oppoſite to the leaſt ſide BC.

Theorem 13. *The external angle of a triangle is equal to the two internal opposite angles.* 32 E. 1, or 10 D. 1.

C.17 Thus, produce AB at pleaſure to D, then we are to prove that the external angle CBD of the triangle is equal to the two internal and oppoſite angles, A and C. To prove this, draw BE parallel to AC, then we have a line DA croſſing two parallel lines, EB, CA; therefore the angle EBD is equal to the angle A, by Theorem 6. Hence we have proved that the part EBD of the external angle is equal to the angle A. It remains now only to be proved, that the other part, viz. CBE, is equal to the angle C, which is done by taking out the angle C, and laying it on the other part of the external angle CBE, it will cover it; conſequently the two internal angles A and C are equal to the whole external angle.

Theorem 14. *The three angles of a triangle taken together, are equal to two right angles, or* 180 *degrees.* 32 E. 1, or 11 D. 1.

For through the vertex C of the triangle ABC, draw a line DE parallel to the ſide AB; and produce the ſides AC and BC at pleaſure, viz. to F and G, then the line FA croſſing two parallel lines DE, AB, the adjacent angles are equal to each other by Theorem 7, 6 therefore, the angle FCE is equal to the angle A; and for the ſame reaſon, the line GB croſſing the two parallel lines, the angle GCD is equal to the angle B; and by Theorem 5, the lines GB and FA, croſſing each other in C, the angle GCF is equal to the angle ACB: But the angles GCD, GCF, and FCE are meaſured by the arcs *dg*, *gf*, and *fe*, which together make half a circle, or 180 degrees; therefore the three angles of the triangle, which are equal to them, muſt alſo be equal to 180 degrees, or two right angles. C.18

Corollary. *Hence if two angles of one triangle be equal to two angles of another, the remaining angle of one muſt be equal to the remaining angle of the other.*

Theorem 15. *If two triangles have two angles of one equal to two angles of the other, each to each, and one ſide of one equal to one* *side of the other, the triangles are equal in every reſpect.*—20 E. 1, or 17 D. 1.

For two angles of one being equal to two angles of the other, the remaining angle of one muſt be equal to the remaining angle of the other by the Corollary to the laſt Theorem. Therefore all the three angles of one are equal to all the three angles of the other, each to each.

C. 12 Hence, if the line DE of the triangle DEF be ſuppoſed to be laid on AB, the triangles will cover each other, and conſequently are equal in every reſpect; but if the young ſtudent ſhould not readily conceive it, he may actually cut out the triangle DEF, and lay it on the other; the line DF will coincide with AC, and EF with BC.

THEOREM 16. *If two triangles have three sides of one equal to the three sides of the other, each to each, theſe triangles are equal in every reſpect.*—8 E. 1, or 17 D. 1.

C.12 For if the triangle DEF be conceived to be, or is actually cut out and laid on the triangle ABC, DE will coincide with AB; the point F will fall on the point C, and the triangles exactly coincide; therefore are they equal in every reſpect.

THEOREM 17. *Similar triangles have their like sides proportional; that is, as any side in one is to the like side in the other, ſo is any other side in the firſt triangle to the like side in the other.*—4 E, 6, or 7 D. 4.

Let DE, EF, and FG be each made equal to AB; the angle D equal to the angle A, and the angles DEH, DFI, and DGK each equal to the angle B; and conſequently, by Corollary to Theorem 14, the remaining angles DHE, DJF, and DKG are each equal to the angle C; and therefore the triangles are all ſimilar. Now if we take the triangle ABC, and lay it on the triangle DEH, it will exactly cover it; again, if we put the triangle ABC on the triangle HNJ, it will exactly coincide with it. Hence it follows, that if DF is twice as long as DE, then DJ will be twice as long as DH or AC, and JF twice as long as JN, or its equal CB. Therefore the ſame proportion that AB has to DF, ſo has AC to DJ, and ſo has CB to JF. Again, if DG is three times as much as AB, DK will be three times as much as AC, and KG three times as much as CB; that is, as AB is to DG, ſo is AC to DK, and ſo is BC to KG; C. 19

the like manner of reaſoning muſt hold good in any other length produced, &c. therefore, in general, as AB, figure 1, is to its like ſide in figure 2, ſo is AC, figure 1, to its like ſide in figure 2, and ſo is CB in figure 1 to its like ſide in figure 2.

Theorem 18. *The ſquares on the two ſhorter ſides of a right-angled triangle taken together, are equal to the ſquare on the longeſt ſide. Or, in other words, that the ſquares on*
C. 40 *the baſe AB and perpendicular BC, viz. the ſquare coloured green, and that coloured yellow, are together equal to the large ſquare on the hypothenuſe, ſtained red.*—47 E. 1, or 27 D. 1.

To prove this in the cleareſt manner, take the two cut parts, which are coloured yellow, and lay them on the reſpective parts of the yellow ſquare, agreeable to their numbers; and they cover the yellow ſquare. In like manner lay the three cut parts ſtained green on the reſpective parts of the ſquare coloured green, and they will exactly cover that ſquare. Now to prove that theſe two ſquares are equal to the ſquare on the hypothenuſe, take all the pieces and lay them on the ſquare ſtained red, agreeable to their reſpective numbers; and they will all together exactly cover that ſquare; and conſequently prove that, the two leſſer ſquares are together equal to the largeſt ſquare. *N. B.* This might have been ſhewn by fewer pieces, but not ſo well adapted to young minds.

Pythagoras on diſcovering this Theorem was ſo fully apprized of its uſefulneſs, that, in gratitude for the diſcovery, he offered up a hecatomb, that is, 100 oxen to the Gods.

BOOK II.

Of Angles in Circles, or parts of Circles.

THEOREM I. *The angle at the center of a circle is double the angle at the circumference; that is, if from any point D in the circumference of the circle two lines DA, DB, are drawn to any two points, A, B, in the circumference, the angle ACB at the center is double that at the circumference ADB.* 20 E. 3, or 10 D. 3. C.20

For if the angle ACB be divided into two equal parts, and either half, viz. ACE or ECB be cut out and laid on the whole angle ADB, it will be found to be equal to it.

THEOREM 2. *The two oppoſite angles of a four-sided figure, inſcribed in a circle, taken*

taken together, are equal to two right angles, or 180 *degrees.*—22 E. 3, or 12 D. 3.

C. 21 For let ABCD be a quadrilateral figure, inſcribed in a circle; then the angle ADC and the angle ABC will together be equal to 180 degrees.

For let the ſide AB be produced at pleaſure to E; on D deſcribe an arc to repreſent the meaſure of the angle D; and with the ſame extent of the compaſſes, placing one point in B, with the other deſcribe a ſemicircle; then, by the figure, the angle ABC with the angle CBE is equal to 180 degrees; cut out the angle D, and laying it on the angle CBE, it will be found to be equal to it; therefore the angle ADC and the angle CBA added together, muſt be equal to the ſemicircle, or 180 degrees.

THEOREM 3. *The angles in the ſame ſegment of a circle are equal to each other.*—21 E. 3, or 11 D. 3.

C. 22 Part 1 & 2d. For take the triangle, part 2d, and placing the vertical angle G on the point C in part 1ſt. ſo that the line GE may fall on the line CA, then will the line GF fall on the line CB, and conſequently the angle ACB will be equal to the angle G; in like manner it will be ſeen that the angle G will alſo exactly cover the angle ADB; therefore the angles ACB and ADB are equal to each other; and wherever we take a point in the circumference, and lines are drawn from it to the extremities A, B, of the chord line AB, the angles will be equal, as may be thus ſhewn: put the Card on a deal-board, and at A and B ſtick in pins; then taking the triangle EGF, lay the angular point G to the pin at A, and the line GF to coincide with the line AB; then keeping the ſides GE and GF always to touch the pins, move the point G from A towards C, D, and B, and the angular point G will always keep in the arc ACDB, and conſequently lines drawn from any point whatever in the arc will form equal angles; and the arc will be the *locus* (or curve) in which the point G will always be found.

THEOREM 4. *The angle in a ſemicircle is a right angle.*—31 E. 3, or 15 D. 3.

The figure, part 1ſt, of this Theorem, C.23 is a ſemicircle, AB being the diameter paſſing through the center, and by the laſt Theorem being a ſegment of a circle, the angles

ADB, AEB, wherever the points D and E are taken in the curve, will be equal to each other. And that they are right angles will appear by taking part the 2d. viz. the right angle FHG, and applying it as we did the triangle EGF in the laſt Theorem.

BOOK III.

Of Plane Superficies.

THEOREM I. *If a line be divided into two equal parts, the ſquare of the line is equal to 4 times the ſquare on half that line.* Corollary 1 of D. 1.

C. 24 Let AC repreſent a line; AB its half; then it is manifeſt by a bare inſpection of the figure, that the ſquare on the whole line AC, viz. ACDE is equal to 4 times the ſquare on AB; viz. equal to the four ſquares *a*, *b*, *c*, *d*.

Corollary. Hence the reaſon of calling the product of two equal numbers a ſquare. For here a line AC is divided into two parts, the ſquare on which containing 4 equal ſquares, the number of which is equal to 2 multiplied by 2; therefore it is called the ſquare of 2; and 3 multiplied by 3 equal to 9 called the ſquare of 3.

THEOREM 2. *If a line be divided into any two unequal parts in C, the ſquare of the whole line is equal to the ſquares on the two parts, and twice the rectangle contained by the parts AC, CB.*—4 E. 2, or 2 D. 2.

For by inſpection of the figure, the ſquare ABDE is the ſquare on the whole line; and is manifeſtly made up of the parts *a*, *b*, *c*, *d*. But *a* is the ſquare on the larger part AC, *b* the ſquare whoſe ſide is equal to the part CB. The rectangle *d* is in length equal to the part AC, and its breadth equal to the part CB; alſo the length and breadth of the rectangle *c* is the ſame as thoſe of the rectangle *d*. Therefore, it will be as in the Theorem. C. 25

THEOREM 3. *Firſt. The diagonal of a rhombus or rhomboides divides it into two equal parts; 2. they are parallelograms; 3. the oppoſite sides and oppoſite angles of parallelograms are equal.*—34 E. 1, or 19 D. 1, and 20 D. 1.

1. If in any rhombus or rhomboides we draw a diagonal AC, it will divide the figure into two triangles; and cutting out the triangle ABC, and laying it on the tri- C. 26

angle ADC, they will exactly cover each other, and therefore are equal in every reſpect; this proves that the *diagonal divides it into equal parts;* which is one part of the Theorem.

2. Again, when the triangle ABC was laid on the triangle ADC, the angles A, B, and C, of the triangle ABC, reſpectively coincided with the angles C, D, and A of the triangle ADC, and conſequently are reſpectively equal; therefore, by Theorem 6, of Book 1, the ſides of the figure are parallel to each other, and hence the *figure is a parallelogram* by the definition, article 40; which is another part of the Theorem.

3. Again, the angle D, having been proved to be equal to the angle B; and part of the angle C, which is diſtinguiſhed by one dot (.) and the other part of the angle C expreſſed by two dots (..) having alſo been proved to be reſpectively equal to the two parts of the angle A; conſequently, the whole angle C is equal to the whole angle A, Hence it is proved that the *opposite angles are equal*, and that the *opposite sides are equal* muſt follow from the triangles having

been proved to be equal in every reſpect; theſe are the laſt conditions which were to be proved.

THEOREM 4. *Every parallelogram, rhombus or rhomboides is equal to a rectangle having the ſame, or equal baſes and equal altitudes.*—This includes 35 and 36 E. 1, or 23 D. 1. and Cor 1, 2, and 3, of D. 23.

For let ABCD be any parallelogram, rhombus or rhomboides, and FECD a rectangle having the ſame altitude or breadth FD, and equal baſes, viz. the baſe AB of the parallelogram equal to the baſe FE of the rectangle; ~~thus~~ then, the triangle AFD is equal to the triangle BEC; for if the triangle BEC be cut out, it will exactly cover the triangle AFD; conſequently, to complete the parallelogram FECD, we take in the triangle BEC, and leave out the triangle AFD which is equal to it; therefore, the parallelogram FECD is equal to the parallelogram ABCD; which was to be proved. C.27

THEOREM 5. *A triangle is equal to half its circumſcribing rectangle having the ſame baſe and altitude, viz. the triangle AEB is equal to half the rectangle ABCD.*—41 E. 1, or 24 D. 1. C.28.

For let the triangle be divided into two parts by letting fall the perpendicular EF (or which is the ſame by drawing it parallel to CB or DA) then cutting out the triangles *d* and *c*, they will reſpectively cover the triangles *a* and *b*; and conſequently the parts of the parallelogram taken out being equal to the whole triangle AEB; the triangle is half the parallelogram ABCD.

THEOREM 6. *Similar ſurfaces are as the ſquares* of their like sides. This Theorem includes not only* 19 E. 1, *for triangles; and* 20 E. 1, *for polygons; but is a general Theorem for all similar ſurfaces whatſoever.*

Two ſquares are ſimilar figures. Let the ſide of the large ſquare be twice as long as the ſide of the leſſer ſquare; then, by a bare inſpection, it appears that the large ſquare contains four of the ſmall ſquares. If AB
C.29 is equal to 1 inch, the leſſer ſquare contains a ſpace of 1 ſquare inch, and if GH is equal to 2 inches, its ſquare contains 2 multiplied by 2, equal to 4 ſquare inches. Hence to ſquare

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* What we call *as the ſquares* (being a more ſimple term) Euclid calls *duplicate ratio*. *Ratio* here means *proportion*.

ſquare a number is to multiply it by itſelf. If the ſide of the large ſquare were 3, it would contain 3 multiplied by 3, equal to 9 times the ſurface, or area of the leſſer ſquare. Hence two ſquares are to one another as the ſquares of their ſides.

If a *triangle* were to be inſcribed in each of theſe ſquares as repreſented by the dotted lines; the triangle inſcribed in the little ſquare would, by Theorem 5, be half of that little ſquare; and the triangle in the large ſquare ~~will~~ would be half of that large ſquare. And therefore, the triangles muſt alſo be as the ſquares of their like ſides; for whatever proportion there is between the whole ſquares, there muſt be the ſame proportion between their like parts. And in general, if a circle or any other figure be inſcribed in a ſquare, and another ſimilar figure in a larger ſquare, though we may not be able to expreſs in numbers, exactly, what part it is of a ſquare; yet, it is manifeſt, whatever part the figure inſcribed in the leſſer ſquare is, of that ſquare, the ſimilar figure inſcribed in the larger ſquare muſt be a like part of that larger ſquare; and therefore, in all ſimilar figures the rule will hold general, that *ſimilar ſurfaces are as the ſquares of their like ſides.*

BOOK IV.

Of Solids.

DEFINITION 1. A *Cube* is a ſolid bounded by ſix equal ſquares, erected perpendicular to each other.*

2. A *Priſm* is a ſolid bounded by ſeveral planes or ends, which may be of any right-lined figure, provided they be equal, ſimilar, and parallel to each other, and the other bounding planes parallelograms. *See ſolids*, No. 10, and 11.

3. A *Parallelopipedon* is a particular priſm, viz. one bounded by ſix parallelograms, whereof thoſe which are oppoſite are parallel and equal. If the baſes are ſquares the piece is called, by workmen, a piece equally ſquared. No. 10.

4. A *Cylinder* is a figure which may be conceived to be generated by the revolution of a right-angled parallelogram about one

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* The ſolids are made in wood, and to prevent miſtakes, have numbers ſtamped on them; thus, there are 8 ſmall cubes numbered from 1 to 8 incluſive, a large cube number 9; and the other ſolids in ſucceſſive order.

of its ſides, which remains fixed; or it may be conceived to be produced by the circle at one end or baſe, moving on parallel to itſelf, till it arrives at the other end. No. 12.

5. From the generation of the cylinder it is manifeſt its baſes are equal, and the circular ends parallel to each other.

6. The *Axis* of a cylinder is an imaginary right line joining the centers of the two baſes or ends; or is that quieſcent line about which the parallelogram revolves.

7. A *Globe* or *Sphere* is a perfectly round ball, and may be conceived to be generated by the motion of a ſemicircle about its diameter; which diameter is called its *axis*, and the center of the ſemicircle is the center of the globe. No. 13.

8. A *Pyramid* is a ſolid contained by a baſe which may be any right lined figure, and as many other planes meeting in one point, as the baſe has ſides. No. 14, and 15.

9. A *Cone* is a ſolid, one end of which is a circle and the other a point; it may be conceived to be generated by the revolution of a right-angled triangle about one of the ſides which contains the right angle; and

the quieſcent ſide of the revolving triangle is the *Axis* of the cone, ſo produced. No. 16.

10. If a *Pyramid* or *Cone* be cut into two parts by a ſection parallel to its baſe, one part will be a pyramid or cone; the other part, viz. that next the baſe, is called the *Fruſtum* of the pyramid or cone. *See Card* 46.

11. A *Solid Angle* is that which is made by the meeting of three or more planes.

12. *Similar Solids* are thoſe which are contained by equal numbers of ſimilar ſurfaces, alike ſituated: Or ſimilar ſolid figures are ſuch as have their ſolid angles equal each to each, and which are contained by the ſame number of ſimilar planes.

THEOREMS.

THEOREM 1. A Pyramid *is one third of its circumſcribing priſm.*—7. E. 12.

THEOREM 2. A Cone *is one third of its circumſcribing cylinder.*—10 E. 12.

In order to prove theſe Theorems mechanically; among the apparatus there is a cylinder, and a cone, of equal heights and baſes, and veſſels (or cups) that will exactly contain them. To prove theſe Theorems mechanically, fill the conical cup with water, and pour it into the cylindrical veſſel; fill the conical veſſel again, and pour it into the cylindrical cup; do the ſame a third time, and the cylindrical veſſel will be filled; conſequently a cone is equal to one third of its circumſcribing cylinder, and it muſt evidently hold good in pyramids, that they are one third of their circumſcribing priſms.

THEOREM 3. A Sphere *or* Globe *is two thirds of its circumſcribing cylinder.* 18 E. 12.

The height and alſo the diameter of the cylinder (in the apparatus) is made equal to the diameter of the globe, as may be tried by a pair of callipers.*

To prove the truth of this Theorem mechanically, put the globe into the cylindrical cup; then if it be two thirds of the cylinder, one third more muſt fill the cup; therefore, fill the conical cup with water, and pour it into the cylindrical cup; then it will, with the globe, fill the cylindrical cup, and conſequently, the globe is two thirds of its circumſcribing cylinder.

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* *Callipers* are bent compaſſes made to take the diameters of round bodies, which cannot be done by a ſtraight pair.

N. B. In Euclid's 12th book, propoſition 8, it is affirmed, that ſimilar pyramids having triangular baſes are one to another in the *triplicate ratio*‡ of that of their like ſides. In propoſition 12, it is aſſerted, that ſimilar cones and cylinders have to one another the triplicate ratio of that which the diameters of their baſes have; and in Theorem 18 it is ſaid that ſpheres have to each other the triplicate ratio of that which their diameters have. He has given ſeparate demonſtrations to each particular Theorem. But all theſe Theorems taken together are not comprehenſive enough; as there may be an infinite number of ſolids which are ſimilar, in which the principle is general; and therefore inſtead of theſe particular Theorems, we ſhall give one that includes all kinds of ſolids whatever.

THEOREM 4. *Similar ſolids of what form ſoever they may be, are in proportion to each other as the cubes of their like ſides, or of any lines in them similarly situated.*

Two cubes are ſimilar ſolids; among the apparatus there is a large cube, each ſide of which is two inches and eight leſſer cubes, each of whoſe ſides is 1 inch. Now 1 multiplied by 1 and that again by 1 is equal to 1, and 2 multiplied by 2 is equal to 4, and 4 multiplied by 2 is equal to eight; which is termed the cube of 8. Therefore, if the large and one of the ſmall cubes be in proportion as the cubes of their like ſides, then the large cube muſt contain 8 of the leſſer, and by putting the 8 leſſer together, they form a cube equal to the large one. Therefore, *cubes are in proportion as the cubes of one of their sides;* or, in Euclid's words, in a *triplicate ratio*. Hence the reaſon why if we multiply a number by itſelf, and that product by the ſame number, the laſt product, in common arithmetic, is called the *cube*.

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‡ What Euclid calls *triplicate ratio* we call *as the cubes.*

Again; If a pyramid be cut out of a large cube it will be one third of that cube; and if out of a leſſer cube, it will be one third of that leſſer cube; and therefore, theſe pyramids muſt be as the cubes of their like ſides; and it is manifeſt, whatever proportion there is between the whole large cube and the whole ſmall one, there muſt be the ſame proportion between the one third of the large, and the one third of the leſs. And generally, if any other ſolid of whatever ſhape be inſcribed in a large cube, and another ſimilar ſolid in the leſs cube, though we may not be able to expreſs exactly, in numbers, what part of the inſcribed ſolid is of the large cube, yet it is a certain part, and the ſimilar ſolid in the leſs cube muſt be a like part of the leſs cube; and conſequently, they muſt be as the cubes of their like ſides, or like lines ſimilarly ſituated.

BOOK V.

Of PROBLEMS.

SECTION I,

Of Problems relating to Angles, Perpendiculars, and Parallel Lines.

Problem I. *To biſect a right line, that is, to divide it into two equal parts.*—10 E. 1, or 6 D. 5.

C. 30 F. 1. About the points A and B, as centers, with any radius greater than half AB, deſcribe parts of circles, viz. the arcs *ab*, *cd*; and through the points of interſection draw the line EF, and it will cut the line AB in G, the point required.

For the arcs *ab* and *cd*, being deſcribed with the ſame extent of the compaſſes, it is manifeſt that, as much of the arc *ab* as reaches beyond the middle of the line from A towards B, ſo much muſt the arc *cd* reach beyond the middle from B towards A; and conſequently the line EF will cut the line AB into two equal parts.

Corollary. This alſo ſhews how to raiſe a perpendicular on the middle of a line.

Problem 2. *To biſect an angle.*—9 E. 1, or 5 D. 5.

Let CAB be the angle which is to be divided into two equal parts. With any radius AD ſweep the arc DE, then with any radius greater than half the diſtance of the two points D, E, about the points D and E, as centers, ſweep the arcs *ab*, *cd*; and through the point of interſection draw the line AF, and it is done, viz. the angle DAF is made equal to the angle EAF. C.30 F. 2.

For the three ſides of the triangle ADF and AEF being equal each to each by the conſtruction, the triangles by Theorem 16 are equal in every reſpect; therefore the angle CAF is equal to the angle BAF, which was to be done.

PROBLEM 3. *To draw a perpendicular on a given line AB, at any given point C in that line.*—11 E. 1, or 7 D. 5.

To do this mechanically, take a little ſquare made in wood, or braſs, like a carpenter's ſquare, C.32 and place it on the line AB at the point C, as repreſented by the ſhaded ſquare, draw the perpendicular CD by the ſide of it, and it is done; if you have not a ſquare, the common little ſcale, which is generally given in a pocket-caſe of inſtruments, will anſwer the purpoſe, for ſmall ſchemes.

Corollary. To know if the ſquare be true, turn it, as repreſented by the unſhaded one C.31 FCD, and then, if both ways will form the ſame right line, the ſquare is true; if not, the angle formed between the two lines will be double the error.

PROBLEM 4. *To draw a perpendicular to a given line AB from any point D, without it.* 12 E. 1, or 8 D. 1.

C.32 Slide the ſquare along the line AB, till it touches the point D, then draw the line EC, and it will be the perpendicular required.

PROBLEM 5. *To draw a line parallel to*

another given line, at a given diſtance.—13 D. 5.

For this purpoſe there is generally (but not always) in a pocket-caſe of inſtruments a parallel ruler. Euclid's method of drawing parallel lines is true in theory but not in practice. The eaſieſt and beſt methods, if you have not a parallel ruler, are thoſe deſcribed in this and the next Problem.

Let AB be the given line, HG the given diſtance; take the diſtance HG in your compaſſes, and ſetting one point in any convenient place C, with the other deſcribe an arc *ab;* then removing the compaſſes, and placing one point at a convenient diſtance from C (the further the more accurate) viz. in D, with the ſame extent deſcribe the arc *cd;* then by the ſide of a ruler draw the line EF to touch but not cut the arcs, and it is manifeſtly the parallel line required. C. 33 F. 1.

Problem 6. *To draw a line through a given point C, parallel to a given line AB.*—31 E. 1, or 12 D. 5.

This is readily done by a parallel ruler, or without it, thus: With your compaſſes take the neareſt diſtance between the point C, and the line AB, which may be known C. 33 F. 2.

by deſcribing the arc *ab;* if it exactly touches the line AB, you have taken a right diſtance; if not, you have taken too little or too much; when you have the true diſtance, placing one point of the compaſſes on a convenient place E, in the line AB, deſcribe the arc *c d;* then from C draw a line CD, to touch the arc *c d;* and it is evidently the line required.

Problem 7. *To make an angle equal to a given angle.*—23 E. 1, or Problem 9 D. 5.

C.34 Let A be the given angle: Draw a line DE; about A and D, with any radius, deſcribe the arcs *ab*, *cd*, then take with the compaſſes the diſtance between the two points *a*, *b*, which ſet off from *d* to *c*, on the arc *dc*, and through the point *c* draw the line DF; then will the angle EDF be equal to the given angle BAC.

Problem 8. *To make an angle equal to any number of degrees.*

For this purpoſe in the pocket-caſes is generally a ſemicircle in braſs, named a protractor, divided into 180 degrees, and numbered forwards and backwards.

C.35 Figure 1 repreſents the manner of laying the protractor in the center at the point C, and the diameter on the line AB; then with the point of a needle make a prick at the number of degrees to be ſet off; then through *d* draw the line CD, and the angle DCB is that required.

But if you have not a protractor, then from the line of chords on your ſcale (which you have in every caſe of inſtruments) take off 60 degrees in your compaſſes, and ſetting one point in C, figure 2, deſcribe the arc *db*, then take off from your ſcale the number of degrees you are to make your angle equal to, and putting one point of the compaſſes in *b*, ſet off on the arc *bd* that extent; draw the line CD, and the angle DCB will be that required.

N. B. Generally there is a needle for pricking off the degrees in the handle of your drawing pen. Surveyors have commonly a whole circle protractor, as being more convenient.

We have now given all the Problems of this nature, that are neceſſary for our preſent purpoſe; and therefore muſt refer thoſe who are deſirous of ſeeing a greater variety of uſeful Problems, with their demonſtrations, to our Geometrician.

BOOK V, Section II.

We ſhall now proceed to give ſome Problems of a different kind, deſigned to ſhew a few inſtances of the uſes of Geometry in taking heights, and diſtances, and other practical purpoſes.

ALTIMETRY,

Or taking Heights of Objects.

Problem 9. *To find the height of an object by the ſhadow of a pole; made either by the ſun or moon.*

Take the board, and ſcrew on the pillar, and put one end of the braſs wire upright in the hole, to repreſent the pole; then, to make it more ſimilar to real practice, place the board in the ſunſhine, or if by night, before a high candle, that the ſhadow both of the pole and pillar may be caſt on the board, which repreſents the ground. Then meaſure the ſhadow of the pole, and the ſhadow of the pillar, alſo the height of the pole, and Theorem 10 comes in to our aid;
C. 36 for let BC in the ſcheme repreſent the height of the pole, AB the ſhadow of the pole, EF

the height of the pillar, and ED its ſhadow; then the triangles ABC and DEF are ſimilar; therefore, if DE be 3 times AB, EF muſt be 3 times the height of BC; or, generally, by the rule of three direct, as AB the ſhadow of the pole is to BC the height of the pole, ſo is DE the ſhadow of the object to EF the height of the object required. *Note,* If neither the ſun nor moon ſhines to caſt a ſhadow, the height of the object may be found by either of the two next Problems.

Problem 10. *To find the height of an object with a pool or baſon of water. (Uſeful when the ſun or moon does not ſhine.)*

Screw on the pillar; and on the board is a ſmall piece of looking-glaſs to repreſent water; take the little carved figure of a man, and conceive him to walk backward or forward, till he ſees the ſhadow of the top of the object in the water; then he is ſuppoſed to meaſure from his feet to the place in the pool where he ſees the image of the top of the object, and from thence to the foot of the object. Let E repreſent the place in C.37 the pool where the image of the top of the object C is ſeen, BC the object, AD the

height

height of the man's eye; then it is well known to all who are acquainted with OPTICS, *that the ray of light CE from the top of the object is reflected to the man's eye at D, making the angle BEC and DEA equal.*[*] Hence the triangles are ſimilar. Therefore, by Theorem 10, if EB be three times the length of AE, then will BC be three times the height of the man's eye. Or generally, ſtate by the rule of three direct: as the diſtance AE is to AD the height of the man's eye, ſo is the diſtance EB to the height of the object BC.

PROBLEM II. *To find the height of an object by a pole. (Of uſe when there is no ſunſhine nor moonlight.)*

Fix up the braſs wire to repreſent the pole; and take the little figure of a man, and carry him backward or forward in a line with the pole and object, till his eye is conceived to ſee the top of the pole and the top of the object in a line; he is ſuppoſed to meaſure from his feet to the pole, and alſo from his feet to the object, and alſo the height of the pole; then Theorem 10 will again aſſiſt us. For if AF repreſent the height of the man's eye, BC the pole, and CD the height of the object; then if we conceive a line FH to be drawn from the man's eye parallel to the ground AC, the triangles FGE and FHD will be ſimilar; then if FH be four times FG, HD muſt be four times GE, to which add CH, (equal to AF,) and we get the required height CD. Generally, as the diſtance the man ſtands from the pole is to the height of the pole above his eye, ſo is the diſtance he ſtands from the object to the height of the object above his eye; to which adding the height of his eye, we get the required height of the object. C.38

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[*] This property of light is alſo applicable to the play of billiards. For if an elaſtic or ſpringy ball (as ivory is) be ſtruck in the direction DE, it will, after ſtriking againſt the ſide of the table, proceed in the direction of the line EC; and therefore the player, before he ſtrikes the ball, is to gueſs where he ſhould direct his ball, ſo that lines conceived to be drawn from the point of contact E, may make the angle DAE equal to the angle CEB.

LONGIMETRY,

Or, taking Diſtances.

Problem 12. *To find the diſtance and poſition of one or more places which are inacceſſible; for example, to find the poſition and*

and diſtance of two churches C and D from A and B.

C.39 At A fix a table with a ſheet of paper on it, and making a mark ⊙ 1 near the corner, take a ruler with ſights on it, and direct it to the towers C, and D, draw a line on the table in that direction, then direct the ruler to B, a pole ſtuck up for that purpoſe, and draw a line in that direction; meaſure the diſtance from A to B, and taking off from a ſcale of equal parts the number repreſenting that meaſured diſtance, ſet it off on the line, and where that diſtance falls make the mark ⊙ 2, to repreſent the ſecond ſtation. Now remove the table to B, having previouſly ſet up a pole at A; and to place it in the ſame direction as before, lay the ruler on the line 2, 1; move the table about till the ruler is in a line with A; then direct the ruler to the places C and D, and draw lines to interſect thoſe drawn when the table was at the firſt ſtation, and the places of interſection *c*, *d*, will ſhew the true relative ſituation of the objects on the table; whoſe diſtance from A and B, and alſo from C to D, are to be meaſured on the table, by the ſame ſcale as you ſet off the diſtance of the ſtations from.

Having already noticed that the 47th of Euclid's firſt book (18th in this eſſay) is looked on as one of the moſt important propoſitions in Geometry, it may reaſonably be expected that we ſhould ſhew an inſtance or two of its uſefulneſs; we ſhall therefore give two problems; one applied to Architecture, the other to Navigation.

PROBLEM 13. *Given the baſe and perpendicular of a right-angled triangle to find the hypothenuſe.*

Example. Given the diſtance between the walls, viz. AB or CD equal to 80 feet, and the perpendicular height of the roof FE equal to 30 feet, required the length of the rafter CE or DE.

In trigonometry it is common to mark given things with a daſh (।) and what is required with a ⊙.

Solution. The triangle CDE, by letting fall a perpendicular, may be divided into two right angled triangles. CD being equal to 80, its half 40 is equal to CF; hence in the right-angled triangle CFE we have given the baſe CF equal to 40, and perpendicular FE equal to 30, to find the hypothenuſe C.41

CE; to do which Pythagoras' moſt valuable Theorem will aſſiſt. For the ſquare on CF may be repreſented by 40 multiplied by 40, equal to 1600, by Corollary to Theorem 1, Book 3, and the ſquare on FE by 30 multiplied by 30, equal to 900. But the ſum of theſe two, viz. 2500 is equal to the ſquare on CE by the 18th Theorem; therefore we have now only to find what number multiplied by itſelf will produce 2500, which is 50; for 50 multiplied by 50 is equal to 2500: therefore the carpenter muſt cut his rafters 50 feet long.

The method of finding this number is called extracting of the ſquare root, and is given in every common book of arithmetic.

In like manner may be found the length of a ladder that will reach the top of a wall, the height of the wall and diſtance of the foot of the ladder from the wall being given.

PROBLEM 14. *Given the hypothenuſe and one of the ſhort sides to find the other.*

This may be applied to navigation.—Example. Suppoſe a ſhip ſails in the ſouth-weſt quarter of the compaſs from an iſland,

and finds by the log-line* that ſhe has ſailed 50 miles, and by an obſervation by a quadrant that ſhe is 30 miles to the ſouthward of the iſland; it is required to find how far ſhe is to the weſtward of the meridian, that paſſes through the iſland.

Let the ſemicircle WSE repreſent half of a mariner's compaſs; I the place of the iſland; S the ſouth, E the eaſt, and W the weſt, C. 42

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* The method made uſe of by mariners to judge of the diſtance ſailed, is to have a line, called a log-line, one end fixed to a piece of a board in form of a quarter of a circle, of about 9 inches radius, (which board is called the log, made by means of lead to ſwim upright) the other end is tied to a reel, on which the line is rolled up, excepting only ſo much as is neceſſary, when the log is thrown overboard, to let it go at a convenient diſtance; at that place a mark is made, and at every 50 feet diſtance is another mark or knot; and it is computed, that as many of theſe knots as run out in *half a minute*, ſo many miles the ſhip ſails in an hour. A quadrant or octant is an inſtrument, by which the angle of altitude of the ſun, at noon, is taken; by which, by the rules in every common book of navigation, they find the latitude of the place the ſhip is in, and thereby how far the ſhip is gone to northward or ſouthward.

⁂ Many years ſince I invented an eaſy and mechanic method of conveying general ideas to Ladies and Gentlemen, how it is poſſible to conduct a ſhip from one country to another.

points. Here we have given the diſtance ſailed IA, equal to 50 miles, and the difference of latitude IB equal to 30 miles, equal to CA, how much the ſhip at A is got to ſouthward of the parallel CE; that is, in the right angled triangle ABI, we have given the hypothenuſe, and one of the ſhort ſides to find the other. The ſquare on IB is equal to 30 multiplied by 30, equal to 900; and the ſquare on AI is equal to 50 multiplied by 50, equal to 2500; by Theorem 18, Book 1, the ſquare on IB, and the ſquare on AB is equal to the ſquare on AI; therefore, if we ſubtract the ſquare on IB equal to 900 from the ſquare on AI equal to 2500, there remains the ſquare on BA, 1600; which is equal to 40 multiplied by 40; conſequently the ſhip is 40 miles to the weſtward of the iſland.

BOOK VI.

Of the Rationalé of the Menſuration of Superficies.

Being an Application of Book III.

Definition. To meaſure any ſuperficies is to find how many ſquare feet, ſquare yards, or any other meaſure it may contain.

Problem I. *To meaſure, or find the area or content of a rectangle, or right-angled parallelogram.*

Multiply the length by the breadth.

For a rectangle may be conceived to be generated by the motion of a line always moving parallel to itſelf. Thus, if AC be conceived to be divided into 3 equal parts (repreſenting 3 feet) and moved on towards DB, when it is come into the poſition 1. 1. it will have deſcribed 3 ſquare feet, equal to 3 multiplied by 1, equal to 3; if into the poſition 2. 2. it will have generated or produced 3 multiplied by 2, equal to 6 ſquare feet, and ſo on; for the number of ſquare feet produced will be always equal to the number of feet in breadth multiplied by the number of feet in length. Hence, when it has paſſed the whole length of feet, or arrived at DB, the product, or number of ſquare feet produced will be 3 multiplied by 7, equal to 21 ſquare feet. If the dimenſions were taken in yards, it would contain 21 ſquare yards. If in Gunter's chains (one of which is equal to 4 poles, or 66 feet in length by which fields are meaſured) then C.43

the field would contain 21 ſquare chains; and to know how many acres; as 10 ſquare chains ~~is~~ are an acre, we muſt divide by 10; and 28 1 divided by 10 is equal to 2 acres and eight-tenths of an acre.

Corollary 1. Hence to meaſure a ſquare, is only to multiply the number of feet or chains, &c. in the ſide by itſelf. As the length and breadth are in ſuch caſe equal.

Corollary 2. Hence, alſo, may appear the reaſon why 3 multiplied by 7 is equal to 7 multiplied by 3; or generally, why *a* multiplied by *b* is equal to *b* multiplied by *a*. For in the annexed figure it will appear, that whether we multiply the number of feet in length by the number in breadth, or the number in breadth by the number in length, it will produce the ſame number of feet, contained in the rectangle.

PROBLEM 2. *To meaſure a rhombus or rhomboides.*

The method is the ſame as in meaſuring a rectangle, viz. to multiply the number expreſſing the length, by the number expreſſing the perpendicular breadth. For, by Theorem 4, Book III, a rhombus or rhomboides is equal to a rectangle of the ſame length and breadth.

PROBLEM 3. *To meaſure a triangle.*

By Theorem 5, Book III, a triangle being equal to half its circumſcribing rectangle, to meaſure it we have only to multiply the baſe by half of the perpendicular, or half the baſe by the whole perpendicular; or take half the product of the whole perpendicular by the whole baſe.

PROBLEM 4. *To meaſure a trapezium.*

A trapezium may be divided into two triangles; therefore run a diagonal line DB, C.44 and let fall the perpendiculars AE, CF; then multiply the diagonal DB by half the ſum of the two perpendiculars AE and CF; or the ſum of the two perpendiculars by half the diagonal; or the whole diagonal by the ſum of both the perpendiculars, and half the product will be the required area or content.

PROBLEM 5. *To meaſure an irregular right lined figure.*

Divide it into triangles, or into trapeziums and triangles; and find their ſeveral contents by problems 3 and 4, and the ſeveral areas added together will give the content of the whole figure.

PROBLEM 6. *To meaſure a regular polygon.*

Every regular polygon may be divided into as many equal triangles as the figure has ſides.

Hence its content may be found by meaſuring the content of one triangle, as ABC, C.45 and multiplying that by the number of ſides (or triangles), the product is manifeſtly the area of the whole; or which is the ſame, and will prepare us for meaſuring a circle, *multiply half the circumference* (viz. half the ſum of all the ſides) *by the perpendicular* CD, let fall from the center on the ſide AB.

N. B. A regular polygon of 5 *ſides* is called a *pentagon* (becauſe it has 5 angles, the meaning of the Greek word, from which the name is derived) and for the ſame reaſon one of 6 *ſides* is called a *hexagon*, (which is the figure drawn on the card); 7 *sides* an *heptagon;* 8, an *octagon;* 9, an *ennagon;* 10, a *decagon;* 11, an *endecagon;* 12, a *dodecagon.*

PROBLEM 7. *To find the circumference of a circle, the diameter being known.*

Every carpenter will tell you that, if the diameter of a circle be 7, the cirumference is 22; and this is ſufficiently exact for common menſuration. Therefore ſtate by the rule of three direct, as 7 is to 22* ſo is the diameter to the circumference nearly. Or more exactly, as 113 is to 355.

PROBLEM 8. *To meaſure a circle.*

It is manifeſt that the greater number of ſides a polygon has, the nearer it approaches to a circle, and conſequently, (with reſpect to practical menſuration) it may be conceived to be a regular polygon of an indefinitely large number of ſides; and therefore, by the laſt problem, its area may be found, by multiplying half the circumference by half the diameter, (as the perpendicular in ſuch caſe is the radius of a circle.)

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* This proportion is generally called *Archimedes proportion*. It was known to *Archimedes* and *Appollonius Pergæus* about 2000 years ago. The proportion of 113 to 355 is generally cited, as the proportion of *Vieta* or *Metius.*——Probably, it will never be found ſtrictly ſpeaking exact. If the diameter is 1. the circumference has been computed to be 3.14159265358979, &c. to more than 100 places of decimals, and even then they could not be affirmed to be exact.

BOOK VII.

Of the Rationality of the Menſuration of Solids.

Being the Application of Book IV.

Problem 1. *To meaſure a cube.*

Cube the number of feet, inches, or yards, &c. which a side meaſures in length.

For inſtance; ſuppoſe the ſide of a cube is equal to 2 feet, then 2 multiplied by 2 is equal to 4; and 4 multiplied by 2 is equal to 8 (the cube of 2) the number of cubic feet it contains. The truth of which is ſhewn by experiment, in illuſtrating Theorem 4 of Book IV.

Problem 2. *To meaſure a ſquare priſm.*

In the apparatus there is a ſquare priſm, marked by lines, to explain this problem. Being a ſquare priſm, the end is a ſquare, and each ſide of the end is divided into two equal parts, which may repreſent 2 feet; and therefore will contain 2 multiplied by 2, equal to 4 ſquare feet, as repreſented by 4 little ſquares. Now, we may conceive this ſolid to be generated by the motion of this ſquare moving in a right line direction, and keeping parallel to itſelf. Hence it is evident, when it has moved on 1 foot, it will have generated 4 cubic feet; if 2 feet, it will have produced 4 multiplied by 2, equal to 8 cubic feet; if it has moved on 3 feet, it will have generated 4 multiplied by 3, equal to 12 cubic feet; and ſo on, till it comes to the end, which in our model is 6 feet; and therefore the whole content will be equal to the area of the baſe, multiplied by the length; in this inſtance 4 multiplied by 6 is equal to 24 cubic feet.

Hence this THEOREM, *to meaſure a ſquare priſm*, ſquare the ſide, and multiply that ſquare by the length.

PROBLEM 3. *To meaſure a parallelopipedon;*

Multiply the breadth, depth, and length together, and the continued product will give its ſolid content.

The reaſon is evident from the method of reaſoning in the laſt problem. For example; let the length be 10 feet, the breadth 4 feet, and depth 2 feet; then the area of the baſe, or end, by problem 1ſt of Book VI, is equal to 4 multiplied by 2, equal to 8 ſquare feet;

and, therefore, when the baſe is conceived to have moved on 10 feet, it muſt have generated 8 multiplied by 10, equal to 80 cubic feet.

PROBLEM 4. *To meaſure a priſm, whatever figure its baſe may be.*

GENERAL RULE. Find the area of the baſe, as ſhewn in Book VI, and multiply that area by the length; the product will find the true content.

The reaſon of this muſt be evident from the method of reaſoning in the two preceding problems.

PROBLEM 5. *To meaſure a cylinder.*

RULE. Find the area of the baſe by the rule for meaſuring a circle, viz. multiply half the circumference by half the diameter; which multiplied by the length will give the ſolid content.

This muſt alſo be evident from the above reaſoning.

PROBLEM 6. *To meaſure a pyramid, whatever figure its baſe may be.*

GENERAL RULE. Find the area of its baſe by the rule for meaſuring that particular figure given in Book VI. which area multiplied by a third of the perpendicular height, will give the content of the pyramid. For, by Theorem 1, Book IV, a pyramid is one third of its circumſcribing priſm.

PROBLEM 7. *To find the ſolid content of a cone.*

RULE. Multiply half the circumference of its baſe by half its diameter, and the product, by problem 8 of Book VI, will be the area of its baſe; which multiplied by a third of the perpendicular height of the cone, will give the ſolid content required.

For, by Theorem 2, Book IV, a cone is one third of its circumſcribing cylinder.

PROBLEM 8. *To meaſure a globe or ſphere.*

By Theorem 3, Book IV, a globe is two thirds of its circumſcribing priſm; therefore, meaſure it as a cylinder, and take two thirds of the product, viz. multiply half the circumference by half the diameter, and the product will be the area of the baſe of the cylinder, which multiplied by the length (height) of the cylinder (which is equal to the diameter of the globe) will give

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the content of the cylinder, two thirds of which will give that of the globe. Or, (which is the ſame thing) multiply half the circumference of the globe by half the diameter, and that product by two thirds of the diameter.

PROBLEM 9. *To meaſure the fruſtum of a cone.*

C. 46 Let BCHF repreſent the fruſtum of a cone, which is to be meaſured. From the greateſt diameter BC ſubtract the leſſer FH; half of which remainder will give BI, or EC, then the triangles BIF and BDA are ſimilar; therefore, ſtate by the rule of three direct; as BI is to IF (the height of the fruſtum) ſo is BD (half the diameter of the cone) to DA, the height of the whole cone, if it were compleated; then, by Problem 7, find the content of the whole cone. If from DA, the height of the whole cone, we ſubtract DG, the height of the fruſtum, we get GA the height of the cone FAH: now find the content of that cone, which ſubtracted from the content of the whole cone, will give the content of the fruſtum BCHF, which was required.

Problem 10. *To find the content of the fruſtum of any pyramid, whatever ſhape its baſe may be.*

General Rule. Find the height of the whole pyramid if compleated, by the method ſhewn in the laſt problem, and then the content of the whole pyramid by problem 6. Again, from the height of the whole pyramid ſubtract the height of the fruſtum, the remainder will be the height of the ſupplemental pyramid; find its content, and ſubtracting it from the content of the whole pyramid, we ſhall manifeſtly find the content of the fruſtum required.

BOOK VII, Section II.

The Application of Geometry to Gauging.

By an Act of Parliament 282 cubic inches is a gallon beer meaſure; and 231 a gallon of wine meaſure.

Hence, the general rule for gauging any kind of veſſel, as the officers of exciſe take their dimenſions in inches, is to find, by the reſpective rule for the figure to be gauged, the number of cubic inches it contains, and divide by 282 for ale or beer gallons, or 231 for wine gallons; but if for malt buſhels, by 2150.

PROBLEM II. *To gauge or find the content or number of gallons in a caſk.*

There are a multitude of rules laid down for Caſk gauging; but the two practical ways, fit for our preſent purpoſe, are,—

C.47 *Method* 1. Meaſure the diameter FG at the middle of the caſk, or bung; alſo the diameter of the head BC; and ſubtract one from the other; and take ſeven-tenths of that difference: (or, which is the ſame, if the young ſtudent has learnt decimals, multiply by .7) which added to the diameter, will give what officers call *a mean diameter;* that is, the diameter of a cylinder, which they ſuppoſe will contain the ſame quantity of liquor as the caſk. In the plate, for the readier conception, the cylinder and caſk, which will contain the ſame number of gallons, are repreſented. The mean diameter then is AE or IH; therefore, multiply half the circumference of the cylinder by half the diameter, and that product by the length of the caſk or cylinder AI, or EH, for the number of cubic inches contained in it; which divided by 282 will give nearly the ale or beer gallons; or by 231, the wine gallons it contains.

N B. If there ſhould be ſome difference between the two head diameters, add them together, and take the half for a mean head diameter, to be uſed in finding the content.

Method 2. By the Gauging Rod.

The Gauging Rod is a rod divided into ale gallons on one ſide, and wine gallons on the other ſide, and has generally inches on a third, and may have areas of circles in ale and wine gallons on the fourth ſide. Its conſtruction and uſe is founded on Theorem 4, Book iv, viz. that ſimilar ſolids are as the cubes of their like dimenſions; but as caſks are not very ſimilar, it is manifeſt, a rod of this ſort cannot be made accurate for diſſimilar caſks. The method of uſing it is ſhewn on the plate, by the line FC, which repreſents the length of the diagonal, which on the rod gives the content; but as the bung may not happen to be exactly in the middle of the caſk, the exciſe officer generally puts it from the bung F to the bottom of the other head B; and, if there is a difference, takes a mean content between the two.

N. B. In large caſks containing valuable liquor, neither of theſe methods ought to be depended on; but a middle diameter ſhould be taken between the bung and one of the heads, alſo between the bung and other head; but theſe methods are too intricate for our preſent purpoſe; which has been to convey ideas of the moſt uſeful and neceſſary propoſitions, in a clear, mechanical manner.

BOOK VIII.

On PROPORTION, &c.

Definitions, &c.

1. When a leſſer magnitude is contained exactly a certain number of times in a greater, it is ſaid to meaſure the greater; and the leſſer, or meaſuring quantity, is called an *aliquot part:* and as magnitudes may be expreſſed by numbers; we may ſuppoſe, for example, one to contain 6℔ weight, the other 3℔; then 3 the meaſuring number is contained twice in 6. Here 3 is an *aliquot part* of 6.

2. A greater magnitude is ſaid to be a *multiple* of a leſſer, when the leſſer is contained exactly a certain number of times in the greater; thus 6 is a multiple of 3, as it contains the leſſer twice, exactly.

3. When magnitudes are compared together, they muſt be both of the ſame kind; as for inſtance, 2lb. weight may be compared with a 6lb. weight; (but 2lb. cannot be compared with 6 yards.) Here, one weight 6lb. is twice as heavy as the other of 3lb.; and therefore 6 is ſaid to be to 3, as 2 to 1; which is called the *ratio*.

4. *Proportion* or rather *Proportionality* is an equality of ratios. Thus, as 6 yards are to 2 yards, ſo are 12 yards to 4 yards. For 2 yards are contained in 6 yards 3 times, and 4 yards are contained in 12 yards alſo 3 times. (Or 6 yards are 3 times 2 yards, and 12 yards are alſo 3 times 4 yards.) Therefore, the *ratio* of 6 to 2 is as 3 to 1; and of 12 to 4 alſo as 3 to 1. Therefore 6, 2, 12 and 4 are ſaid to be in *direct proportion*, and uſually written thus, as 6 : 2 :: 12 : 4; or generally, if a, b, c, d, any four magnitudes or numbers are in direct proportion, we write,

as *a* : *b* :: *c* : *d*, which we read thus; as the quantity or number repreſented by *a* is to that denoted by *b*, ſo is the quantity or number repreſented by *c* to that repreſented by *d*.

5. Three numbers are ſaid to be proportional, when the *firſt* is to the *ſecond* as the *ſecond* is to the *third;* that is, for inſtance, as 2 : 4 :: 4 : 8; or generally, as *a* : *b* :: *b* : *c*.

6. If any nnmber be multiplied by itſelf, the product is called its *ſquare;* thus 9 is called a ſquare number; for 3 multiplied by 3 is equal to 9. See explanation of Theorem 6, Book III.

7. The *ſquare root* of any number, is a number which multiplied by itſelf will produce the given number; thus 3 is the ſquare root of 9, for 3 multiplied by 3 is 9.

8. The ſquare root of the product of any two numbers is called a *geometrical mean.*

9. Half the ſum of any two numbers is called an *arithmetical mean.*

10. If any number be multiplied by itſelf, and that product by the ſame number, this

laſt product is called the *cube* of the given number; thus 3 multiplied by 3 is equal to 9, and 9 multiplied by 3 is equal to 27; which is therefore called a *cube number*. See explanation of Theorem 4, Book IV.

11. The *cube root* of any number is a number, which multiplied by itſelf, and that again by the ſame number, will produce the given number; thus 3 is the *cube root* of 27; for 3 multiplied by 3 is equal to 9, and 9 multiplied by 3 is equal to 27.

12. If a number does not admit of an exact root, it is called a *ſurd* or *irrational number*.

Note, The method of extracting the ſquare root, and cube root, of any number may be ſeen in any common book of arithmetic.

Before we proceed farther, the young ſtudent ſhould make himſelf acquainted with the following characters; which are not only neceſſary, for underſtanding the doctrine of proportion here treated of; but, alſo, for reading moſt mathematical works.

CHARACTERS EXPLAINED.

∵	Therefore.
$+$	The ſign of addition; thus $2+3$, ſignifies that 2 and 3 are to be added together; and is read 2 *plus* 3; or 2 more 3; or 2 and 3.
$-$	The ſign of ſubtraction; as $3-2$ denotes that 2 is to be taken from 3; read thus, 3 *minus* 2; or 3 leſs 2.
$\times$	The ſign of multiplication; as 3×4 ſtands for 3 multiplied by 4.
$\div$	The ſign of diviſion; as $6\div2$ ſignifies that 6 is to be divided by 2; ſometimes written thus, $\frac{6}{2}$. And $\frac{a+b}{2}$ ſignifies that the numbers denoted by *a* and *b* are to be added together, and the ſum divided by 2.
$=$	The ſign of equality; as $3+2=5$, is to be read 3 plus 2, or 3 and 2 are equal to 5.
$\surd$	Is called the radical ſign, and ſignifies the ſquare root; thus $\sqrt{4}$ is to be read, the ſquare root of 4; $\sqrt[3]{8}$ denotes the cube root of 8. $\sqrt{a\times b}$ ſhews that the number repreſented by *a* is to be multiplied by the number repreſented by *b*, and the ſquare root of that product to be extracted.

As the relation of magnitudes may be repreſented by numbers, we ſhall, for the ſake of clearneſs, expreſs them by numbers; or by letters, to repreſent them more generally.

THEOREM 1. *When four numbers are in direct proportion, the product of the extremes is equal to the product of the means.* 16 E. 6, or 6 D. 4.

Thus, as $6:2::12:4$, by Definition 4. Here the product of the extremes is $6\times 4 = 24$; and 2×12, the product of the means, is alſo $=24$. Or generally, if $a:b::c:d$, then $a\times d = b\times c$.

THEOREM 2. *If four numbers are in direct proportion, the fourth term is equal to the product of the ſecond and third, divided by the firſt.*

For example; if $6:2::12:4$, then $2\times 12 = 24$, and $24 \div 6 = 4$, the fourth number. Or generally, as $a:b::c:d$; then $b\times c$, and $\div a = d$; which is more frequently expreſſed thus, $\frac{b\times c}{a} = d$.

N. B. On this Theorem, *the rule of three direct*, in common arithmetic, is founded.

Corollary. (16 E. 1, or 6 D. 4.) Hence, if ~~the product of~~ any four numbers, expreſſed generally by a, b, c, d, have the product of the extremes $a\times d$, equal to the product of the means $b\times c$; theſe four numbers are proportional, that is, as $a:b::c:d$.

Euclid and other Geometricians frequently make uſe of proportions, which they quote by the terms, by *permutation* or *alternately*, by *inversion*, by *composition*, by *division*, by *conversion;* which will be explained in the following Theorems.

THEOREM 3. (11 D. 4.) *If four numbers are proportional, for inſtance, as* $6:2::12:4$; *or generally, as* $a:b::c:d$, *they will alſo be proportional when taken* inverſely; *that is, it will be, as* $2:6::4:12$; *or generally, as* $b:a::d:c$.

For the product of the extremes is $=2\times12=24$; and the product of the means $=6\times4=24$, alſo. Or, $b\times c=a\times d$, agreeable to Theorem 1.

THEOREM 4. (16 E. 1, or 12 D. 4.) *If four numbers are proportional, they will be ſo* alternately; *that is, if when direct,* $6:2::12:4$,

::12:4, *or generally,* $a:b::c:d$; *then it will be alternately,* as 6:12::2:4; *or generally, as* $a:c::b:d$.

For in ſuch caſe, the product of the extremes is $=6\times4=24$; and of the means, $12\times2=24$, alſo; or $a\times d=c\times b$: ∵ they are *alternately proportional;* that is, as 6:12::2:4, as is expreſſed in the Theorem; or generally, as $a:c::b:d$.

THEOREM 5. (18 E. 5, or 13 D. 4.) *If four numbers are proportional, they will alſo be compoundedly proportional; that is, for example, if* 6:2::12:4; *or generally, as* $a:b::c:d$; *then it will be compoundedly, as* $6+2:2::12+4:4$; *or generally, as* $a+b:b::c+d:d$.

For $6+2$ being $=8$, and $12+4=16$; it is, as 8:2::16:4; for the product of the the extremes, $8\times4=32$, and the product of the means, 16×2 is alſo $=32$; ∵ 8, 2, 16, and 4, are proportional; that is, as $6+2:2::12+4:4$; or generally, as $a+b:b::c+d:d$. As in the Theorem.

THEOREM 6. (17 E. 1, or 14 D. 4.) *If four numbers are proportional, they will alſo be pro-*

proportional by diviſion*; *that is, for inſtance, if* $6:2::12:4$; *or generally, if* $a:b::c:d$; *then it will alſo be, as* $6-2:2::12-4:4$; *or generally, as* $a-b:b::c-d:d$.

For $6-2$ being $=4$, and $12-4=8$, the Theorem is, as $4:2::8:4$; for the product of the extremes $=4\times4=16$; and the product of the means 2×8 is alſo $=16$; $\therefore$ $6-2:2::12-4:4$; or generally, as $a-b:b::c-d:d$; as in the Theorem.

THEOREM 7. (15 D. 4.) *If four quantities are proportional, they will alſo be proportional by* converſion; *that is, if* $6:2::12:4$; *or generally, as* $a:b::c:d$; *then will* $6:6-2::12:12-4$; *or as,* $a:a-b::c:c-d$.

For $6-2$ being $=4$; and $12-4=8$; the analogy is, $6:4::12:8$. The product of the extremes, $6\times8=48$; and the product of the means, 4×12 is alſo $=48$; $\because$ 6, 4, 12, and 8, are proportional; that is, $6:6-2::12:12-4$; or, $a:a-b::c:c-d$. As in the Theorem.

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* *Diviſion* here does not mean an arithmetical diviſion; but only, that the line or number, &c. is ſuppoſed to be divided into two parts; and were it not for deviating from Euclid's Terms, I ſhould rather have ſaid, by *Subtraction*.

THEOREM 8. (16 D. 4.) *If four numbers are proportional, they will alſo be proportional* mixtly; *that is, if* $6:2::12:4$; *or generally,* $a:b::c:d$; *then will* $6+2:6-2::12+4:12-4$; *or generally, as* $a+b:a-b::c+d:c-d$.

For $6+2=8$; $6-2=4$; $12+4=16$; $12-4=8$; then, by the Theorem, the analogy is $8:4::16:8$. The product of the extremes is $=8\times8=64$; and the product of the means $=4\times16=64$ alſo; $\because$ $6+2:6-2::12+4:12-4$. Or, generally, $a+b:a-b::c+d:c-d$; as in the Theorem.

THEOREM 9. (17 E. 6, or 17 D. 4.) *If three numbers are proportional, the product of the two extremes is equal to the ſquare of the means; thus, for inſtance,* 2, 4, 8, *are three proportional numbers, by* Definition 5; *for* $2:4::4:8$; *or generally, if* a, b, c, *denote three proportional numbers, that is, if* $a:b::b:c$, *by* Definition 5, *they are proportional; then we affirm, that* $a\times c$, *the product of the extremes, is* $=b\times b$, *the ſquare of* b, *the mean number.*

For if $2:4::4:8$, then the product of the extremes is $=2\times8=16$; and 4×4, the ſquare of the mean is alſo $=16$. Or generally, if $a:b::b:c$, then the product of the extremes $a\times c=b\times b$, the ſquare of b, the mean.

Corollary. Hence, if $2:4::4:8$, the ſecond term 4, by Definition 5, is a geometrical mean, the ſquare of which, by this Theorem, is $=2\times8=16$; and conſequently, $\sqrt{2\times8}=4$ is that mean; or generally, b, the geometrical mean between any two numbers, expreſſed by a and c, is $=\sqrt{a\times c}$. But, by Definition 9, the arithmetical mean of 2 and 8 is $=\frac{2+8}{2}=\frac{10}{2}=5$; or generally, between a and $b=\frac{a+c}{2}$. So that the geometrical mean between 2 and 8 is $=4$; but the arithmetical mean $=5$.

From reviewing the preceding Theorems, we may draw up, as it were, a PROPORTIONAL SPECULUM*, which may be of the greateſt uſe to young Students.

If

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* I have made uſe of the term SPECULUM above; becauſe, as a perſon can ſee the whole of his face at *once* in a glaſs, ſo here, at a ſingle glance, he may ſee at one view, the principal propoſitions which are contained in Euclid's very difficult vth. Book, and ſome of the vith; with ſome uſeful theorems not in that Author. For I have purpoſely condenſed the theorems as

The SPECULUM.

If 1. $a:b::c:d$, in direct proportion, then $a\times d=b\times c$, by Theorem 1.

2. $d=\frac{b\times c}{a}$. Theorem 2.

3. $b:a::d:c$, called *inverſely*. Theorem 3.

4. $a:c::b:d$, *alternately*. Theorem 4.

5. $a+b:b::c+d:d$, *compoundedly*. Theorem 5.

6. $a-b:b::c-d:d$. By *diviſion*. Theorem 6. (See the note.)

7. $a:a-b::c:c-d$, By *conversion*.—Theorem 7.

8. $a+b:a-b::c+d:c-d$. *Mixtly*. Theorem 8.

9. If 3 numbers, a, b, c, are in proportion, viz. as $a:b::b:c$, then $b\times b=a\times c$, Theorem 9.

10. The geometrical mean between any two numbers a and c, is $=\sqrt{a\times c}$. Corollary, Theorem 9.

11. The arithmetical mean between a and c is $=\frac{a+c}{2}$. Corollary, Theorem 9.

as it were, into a much leſs ſpace than in any treatiſe, that has come to my hands; by giving barely the theorems, without any mixture of demonſtrations. Many years ſince I was ſo ſenſible of the importance of ſuch views, that it induced me to give in my *Geometrician*, *a Speculum*; in which the theorems are ſo regularly claſſed, as to enable a young ſtudent, when in want of one, readily to find a proper theorem to anſwer his particular purpoſe, without being obliged carefully to hunt or turn over Euclid, leaf by leaf, from one end to the other.

N. B. If the ſtudent is deſirous of ſeeing this ſubject more fully treated of, and more ſtrictly demonſtrated, he may conſult my Elements of Geometry; in which the theorems are demonſtrated in an eaſier manner, than Euclid's method would admit of.

CONCLUSION.

We have now gone through what was propoſed, viz. to deliver the principal or more valuable Propoſitions of Geometry, and ſhewn their truth, for the moſt part, by *mechanical proofs;* and where theſe were not convenient, as particularly, in the doctrine of Proportion in this VIIIth Book, by the moſt ſimple and The eaſieſt method that could be deviſed.

Though the knowledge of Geometry, conveyed by this method, may be ſufficient for moſt perſons, yet if the Author could prevail, he would adviſe every one, deſirous of acquiring a habit of thinking and reaſoning with preciſion, not to reſt here; but to ſtudy a more ſtrict method of demonſtration, as given in Euclid or in ~~my~~ his Geometrician;

metrician: For it has been allowed by Mr. *Locke*, Dr. *Barrow*, and other proper judges, that, following the ſtrict geometrical method of demonſtration will conduce, to our acquiring the true art of reaſoning, more than all the ſyſtems of *Logic** of the ſchools. It is not indeed neceſſary that all men ſhould be profound mathematicians; but it is commendable, it is neceſſary, to acquire that way of reaſoning, to which Geometry neceſſarily habituates the mind.

" Would you have a man reaſon, (ſays Mr. *Locke*, in his Eſſay on human underſtanding) you muſt uſe him to it betimes; exerciſe his mind in obſerving the connection of ideas, and following them in train. Nothing does this better than the mathematics; which therefore, I think, ſhould be taught all thoſe who have time and opportunity, not ſo much to make them mathematicians as to make them reaſonable creatures; for, though we all call ourſelves ſo, becauſe we are born to it, if we pleaſe, yet, we may truly ſay, nature gives us but the ſeeds of it. We are born to be, if we pleaſe, rational creatures, but it is uſe and exerciſe only that make us ſo; and we are, indeed, ſo no farther than induſtry and application have carried us." In

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* Mr. *Locke* thought ſo little of *Rhetoric* and *Logic*, in conducing to make a good reaſoner, that in his *Eſſay on Education*, he ſays, " I have ſeldom or never obſerved any one to get the ſkill of reaſoning well, or ſpeaking handſomely, by ſtudying thoſe rules which they pretend to teach; and therefore I would have a young gentleman take a view of them in the ſhorteſt ſyſtems could be found, without dwelling long on the contemplation and ſtudy of thoſe formalities. Right reaſoning is founded on ſomething elſe than predicaments and predicables, and does not conſiſt in talking in mode and figure itſelf."—If you would have your ſon ſpeak well, " Let him be converſant in *Tully*, to give him a true idea of eloquence; and let him read thoſe things that are well writ in *Engliſh*, to perfect his ſtyle in the purity of our language. Be ſure not to let your ſon

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ſon be bred up in the art and formality of diſputing, either practiſing it himſelf, or admiring it in others; unleſs, inſtead of an able man, you deſire to have him an inſignificant wrangler, opiniatre in diſcourſe, and priding himſelf in contradicting others; or, which is worſe, queſtioning every thing, and thinking there is no ſuch thing as truth to be ſought, but only victory in diſputing. Truth is to be found and maintained by a mature and due conſideration of things themſelves, and not by artificial terms and ways of arguing, which lead not men ſo much into the diſcovery of truth as into a captious and fallacious uſe of doubtful words; which is the moſt uſeleſs and diſingenuous way of talking, and moſt unbecoming a gentleman, or a lover of truth, of any thing in the world."

IN my Geometrician, for the amuſement of young Students, I gave a few Paradoxes; and, with the ſame deſign, ſhall here give two additional ones.

Paradox 1. A line may be conceived continually to approach nearer and nearer to another line, and yet, if infinitely continued would never meet.

Paradox 2. A Gentleman has four rooms, with a hole in each window-ſhutter; and, for making experiments, he has frequently occaſion to darken ſometimes one room, ſometimes another; and though the holes in the ſhutters are of different forms, viz. in one the hole is a circle, in another a triangle, in the third a ſmall oblong, and in the fourth a larger oblong, yet he has but one piece of wood, ſo formed as to fill or cloſe either hole, as occaſion may require.

If, after ſome conſideration, the young Pupil does not diſcover the poſſibility of theſe Paradoxes, he may then, but ſhould not before, read the following explanations.

Solution to Paradox 1. An *Aſſymptote* to an *Hyperbola* will anſwer this Paradox, as is ſhewn by the Writers on *Conic Sections*; but that is a ſubject which requires too much mathematical knowledge, to be introduced in this Eſſay, therefore, take the following more eaſy ſolution, viz. Let AC and BD be two right lines, perpendicular C.48 to each other. From the point A draw any number of lines *Am, Am,* &c. at pleaſure. Take the diſtance BC in your compaſſes, and ſet off that diſtance from the places of interſection of the ſeveral lines, with the line BD, viz. from *c* to *m*, *c* to *m*, &c. then with a ſteady hand join the ſeveral points *m, m, m,* &c. and the curve line CE will be that required. For that the curve line CE approaches nearer and nearer to the line CD, as the points *m, m,* &c. become more remote from the line AC, appears from a bare inſpection of the figure; the perpendiculars *mn, mn,* &c. decreaſing: And that, if infinitely continued it would never meet the line BD, may be eaſily underſtood, by conſidering that the point A being above the line BD, the line *Am,* however far off from the point B, muſt interſect and form an angle *mcn*; and *ncm* will be a right angled triangle; and conſequently, however ſmall the triangle is, it muſt have a perpendicular *cm,* which though indefinitely ſhort, muſt be a real diſtance; and conſequently, the point *m* can never coincide with the point *n*; and therefore the lines BD and CE can never touch.

N. B. In the Paradox it is ſaid, A line may be *conceived* to approach continually; becauſe it cannot be actually drawn, as the lines would then have ſome breadth, and conſequently, when the perpendicular *mn* becomes leſs than the breadth of the lines, they would run into each other.

PARADOX 2, explained. Among the apparatus is a piece of mahogany, like a ruler, with holes, as deſcribed in the Paradox; and alſo a piece of wood in form of a wedge, with a circular baſe. It is manifeſt, the baſe being a circle will fill a round hole; if the piece be put in the flat way, it fills the large oblong hole; if ſideways, it will fill the triangular hole; and if put in as a wedge, it will ſhut up the ſmall oblong hole.

Note to PROBLEM 9. BOOK VII.

The practical rule generally made uſe of for meaſuring the fruſtum of a cone is—*To the product of the top and bottom diameters add one-third part of the ſquare of their difference, and multiply the ſum by the height, and that product by .7854 will give the content*; in cubic feet, if the dimenſions were taken in feet; but if in inches, as is always done in gauging, the content will be cubic inches; which divided by 282 will give the content in ale gallons; if by 231, in wine gallons. But for the purpoſe of gauging round veſſels, inſtead of multiplying by .7854, and then dividing by 282 or 231, it is more expeditious to omit multiplying by .7854, and make uſe of the diviſor 359 for ale, or 294 for wine gallons.

This rule is deduced from the rationalé given in the problem; but by a method too foreign to our preſent deſign, to be here introduced.

Note to BOOK V. SECTION II.

In problems 9, 10, and 11, ſeveral methods of taking heights are ſhewn, built on the doctrine of ſimilar triangles. But if a tower or any building is erected on an horizontal plane, its height may be found independent of ſimilar triangles, and without any quadrant or other inſtrument for taking angles, by the following ſimple method.

On a piece of board draw two lines, viz. one perpendicular to the other. Stick two pins upright on one of the lines, as far diſtant from each other as conveniently may be, biſect the right angle, and at the angular point hang a plumb-line; then ſtanding at a diſtance (as nearly as you can gueſs equal to the height of the object) hold the board ſo that the two pins may be in the direction

of the top of the object; and obſerve if the
plumb-line falls on the biſecting line; if not,
walk backward or forward till it does; then will
Card the angle HFD and FDH be equal, and con-
38 ſequently, if we meaſure from A to C (which is
equal to FH) the diſtance will be equal to HD;
to which adding HC equal to FA, the height of
the man's eye, we ſhall have CD, the height of
the object required.

N. B. The methods ſhewn in problems 9 and 11 will hold good when the object is on a regular aſcent or deſcent, as well as if on an horizontal plane, as the triangles will in ſuch caſe be alſo ſimilar.

Though in p. 84, I recommended ſtudying the ſtrict method of demonſtration uſed by the ancients; yet I would be underſtood as recommending it only in a degree. For, if a young man were obliged to ſtudy Euclid thoroughly, it would be too great a waſte of time, which might have been more uſefully employed; for though at the time Euclid wrote, any perſon who underſtood his elements would be accounted a good mathematician, yet now theſe muſt only be conſidered as an introduction to other branches of ſcience. It certainly would be better, after having acquired a competent knowledge of the moſt uſeful propoſitions with their ſtrict demonſtrations, to apply them to practical purpoſes; to mixt mathematics; to enquire into the works of nature and art. And here a very extenſive and delightful field opens to view; fully ſufficient to employ his underſtanding, to make him a more uſeful member of ſociety, and enable him to ſee more of the goodneſs, wiſdom, and power of GOD, in the creation and preſervation of all things.

THE END.

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* Now out of Print.—See Page 91.